Wireless Networking & Communications Group

THE UNIVERSITY OF TEXAS AT AUSTIN
RADIONAVIGATION LABORATORY

TEXAS
The University of Texas at Austin

Augmenting GPS with PNT from LEO

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Work supported by Sandia National Laboratories, US Army Futures Command

Call to Action for GPS

- Protect
- Toughen
- Augment

LEO

- DARPA: Blackjack
  - "Pivot to LEO"
  - Comms+PNT payloads (in phase 2)
  - Government-owned vehicles ($4-15 million per SV)
- DLR + GFZ Potsdam: Kepler
  - Next-generation Galileo
  - Space-based atmosphere-free global synchronization + precision orbit determination

Commercial LEO

OneWeb Satellites
A **OneWeb** and **Airbus** joint venture

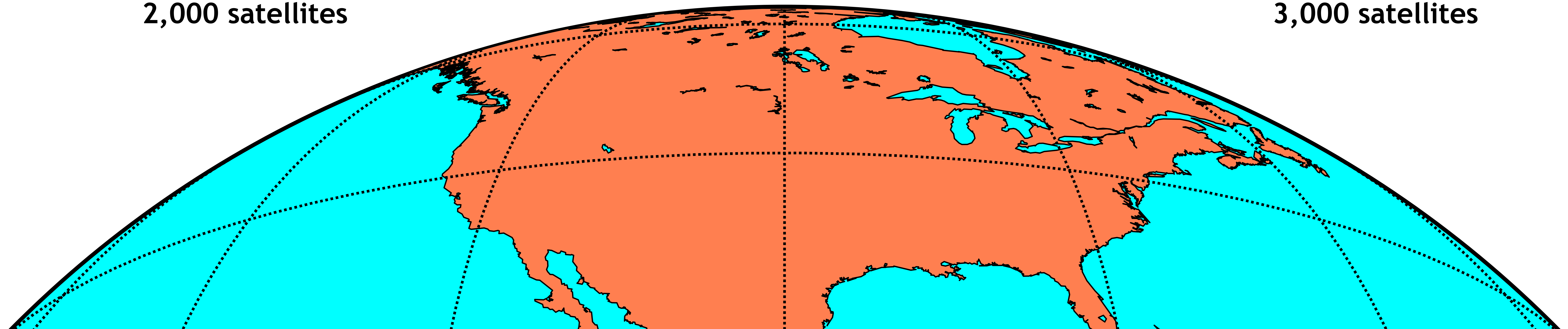
2,000 satellites

STARLINK

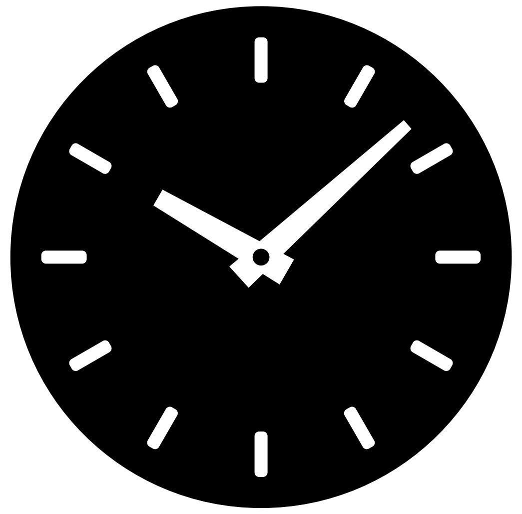
12,000-30,000 satellites

Kuiper Systems
amazon

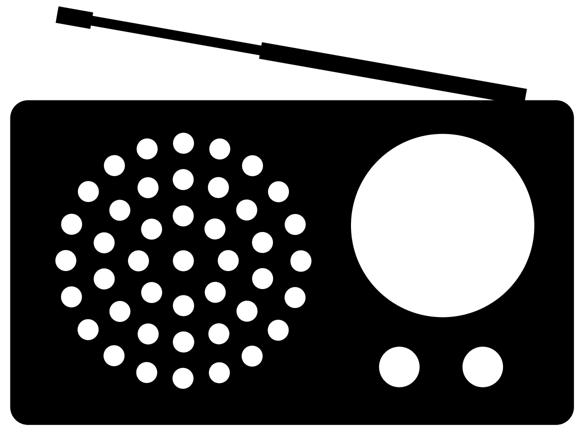
3,000 satellites



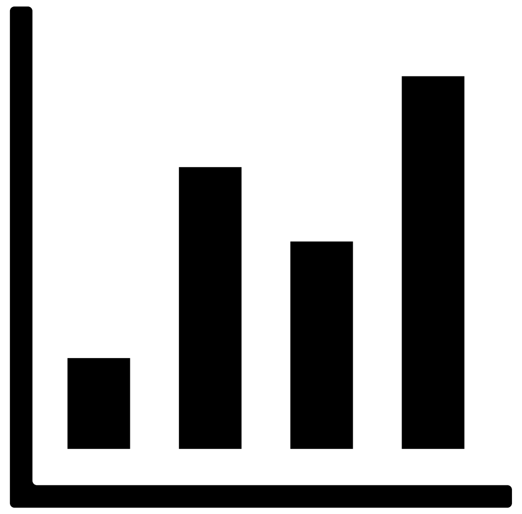
Elements of Fused PNT



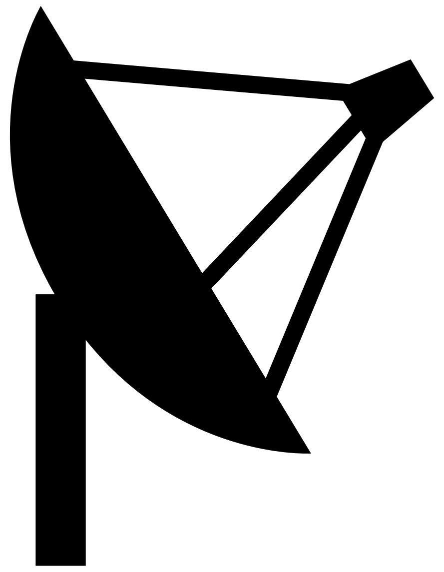
Clock



Radio



Spectrum



Antenna

Already present … just not purpose-built!

Q: Can we compromise on clock quality?

A: Yes. Discipline to GPS and update frequently. (e.g. 50s)

Q: Can we compromise on signal structure?

A: Yes. High SNR, wide bandwidth.

Q: Can we share spectrum, time, and antennas with (primary) data customers?

A: Yes. TDMA+FDMA, high SNR. Service cost per 20km spot beam ≈ cost of 1 MB/s downlink bandwidth

Prior work

- High-Integrity GPS program (Iridium, Boeing, Coherent Navigation, 2007-2011)
  - Now Satelles
  - Not single-epoch (Transit-style positioning)
- Tyler Reid, A. Neish, P. Enge (2016-2018)

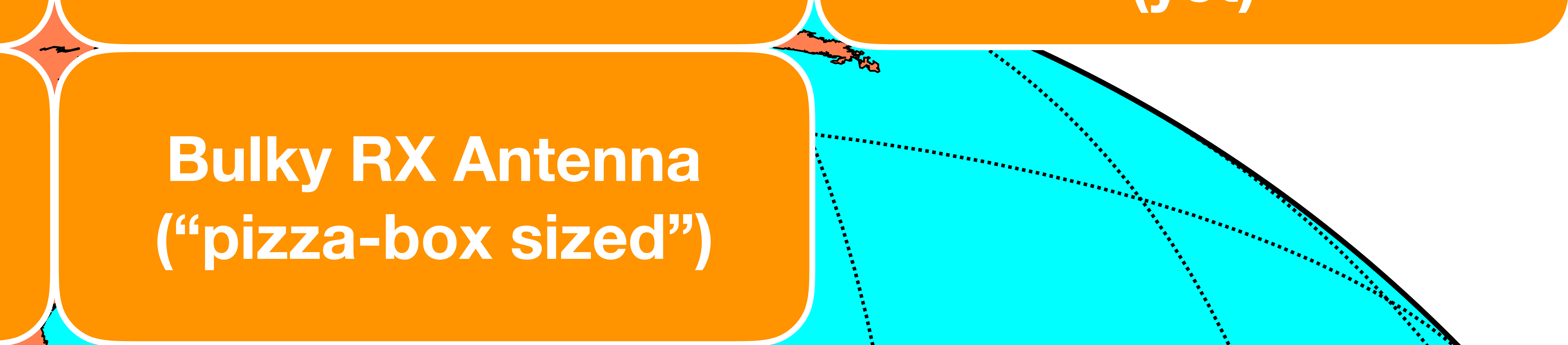

GPS
20,000 km
Commercial Broadband LEO
400 km

GPS
20,000 km
Commercial Broadband LEO
400 km

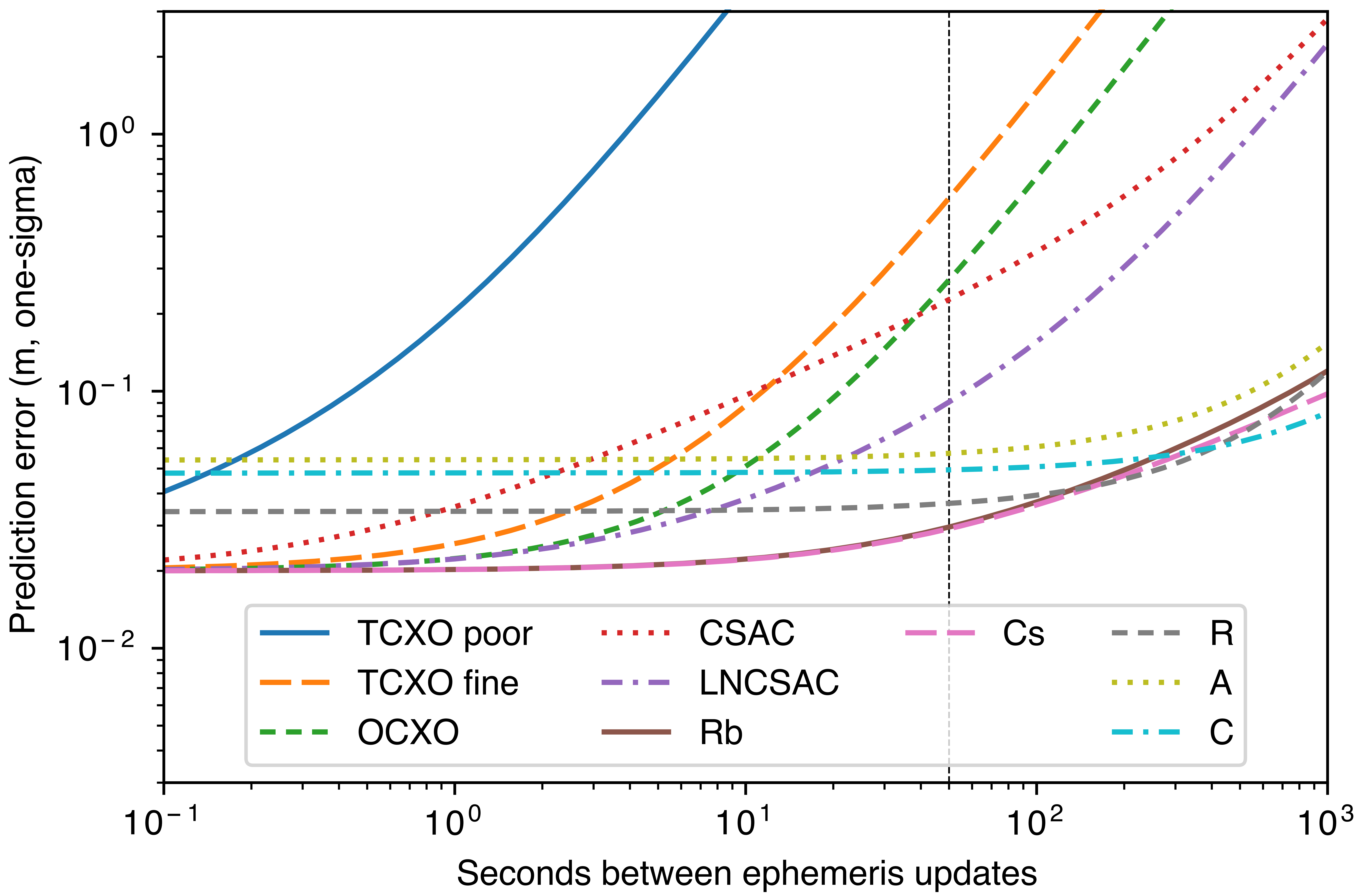

GPS
20,000 km
+31 dB
Power Flux Density
+25 dB
RX antenna selectivity
(vs. horizon)
20 ms
Single-Epoch
Positioning and Timing
1,000×
More satellites
700×
Faster Multipath
Decoherence
30×
Greater Bandwidth
(vs. civilian signals)
4,000,000×
Bandwidth for Data
(Zero-lag Ephemerides)
No atomic clocks
(OCXO sufficient)

Bottom Line

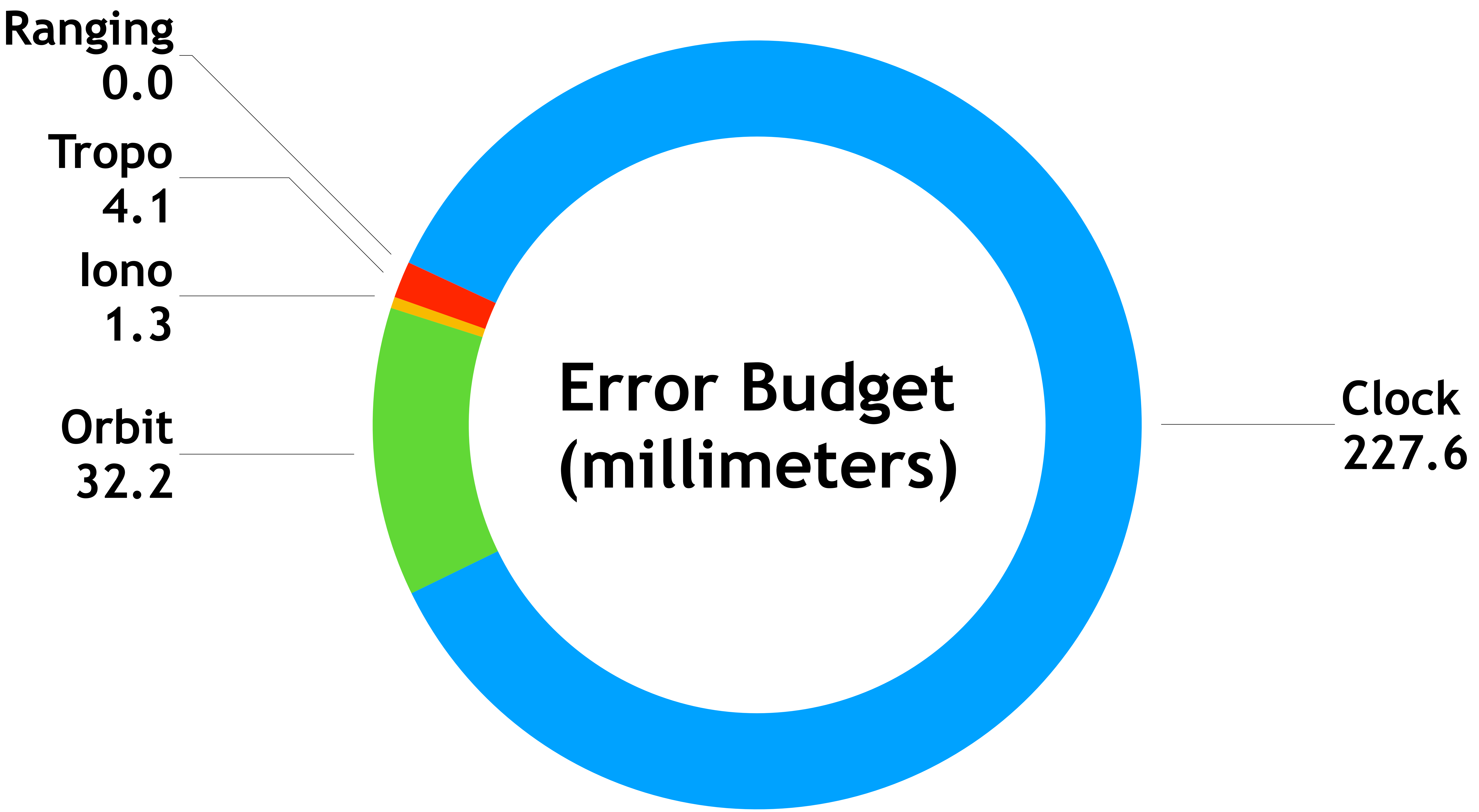
+56 dB Anti-Jam	95% user error: 32cm (AoE 1s) 1.21m (AoE 50s)	Hardware costs to US Govt: $0.00
Reliant on Commercial Service Providers	Pay-as-you-go (cf. Iridium)	Not resilient to global GPS Outage (yet)
Intermittent Ranging	Bulky RX Antenna (“pizza-box sized”)	



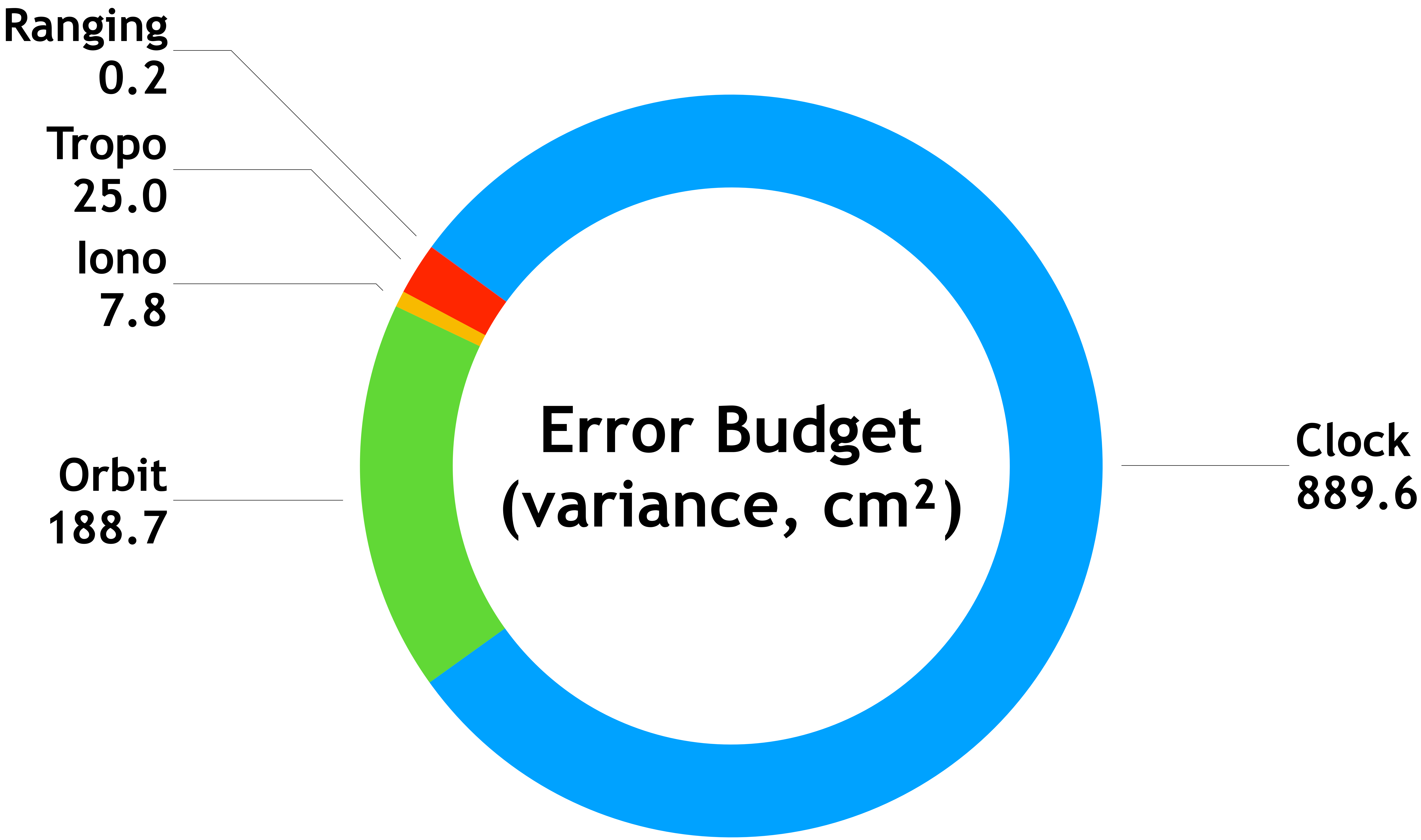
Back-up Slides



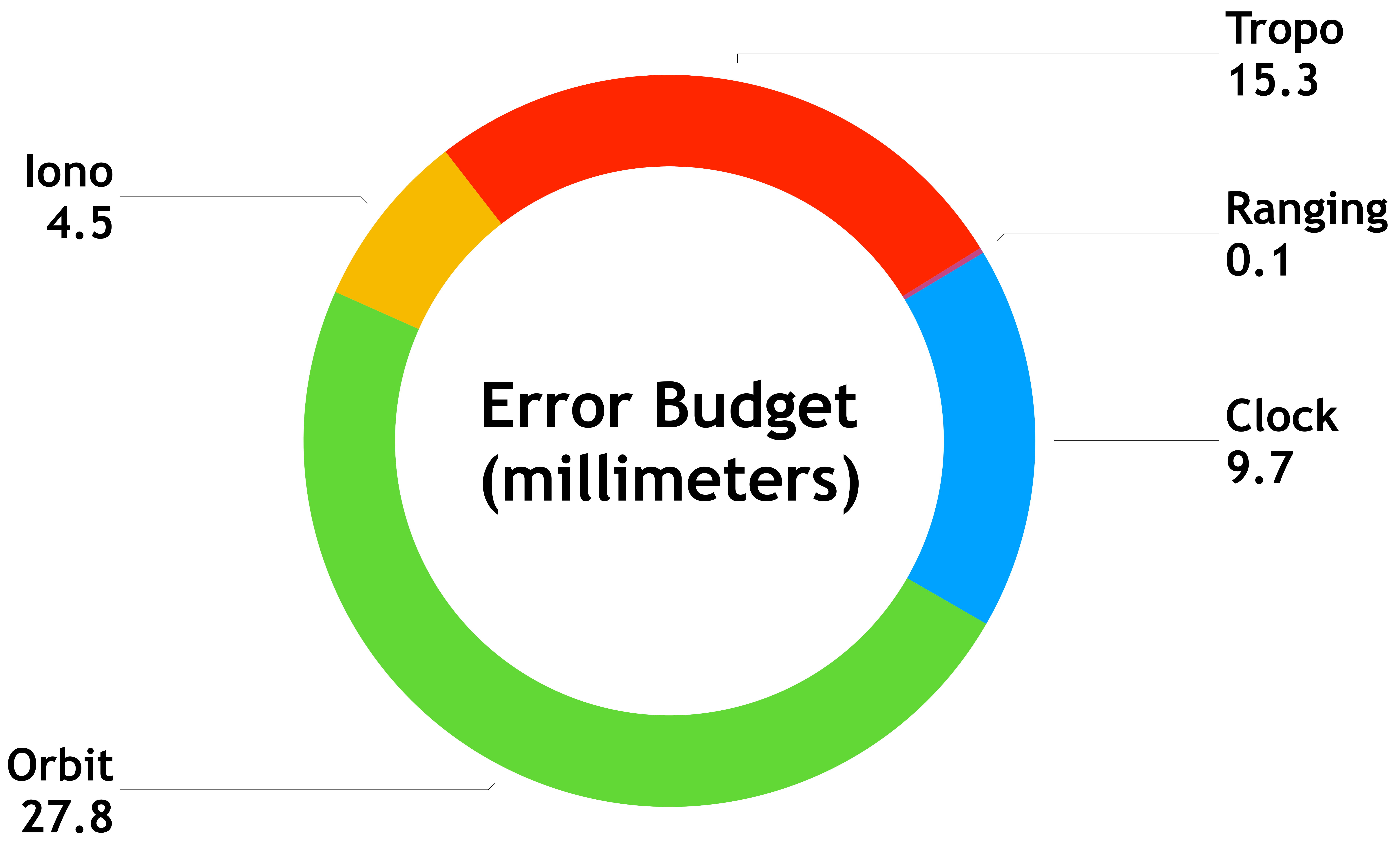
Prediction error (m, one-sigma)
10^{0}
10^{-1}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Seconds between ephemeris updates
TCXO poor
TCXO fine
OCXO
CSAC
LNCSAC
Rb
Cs
R
A
C



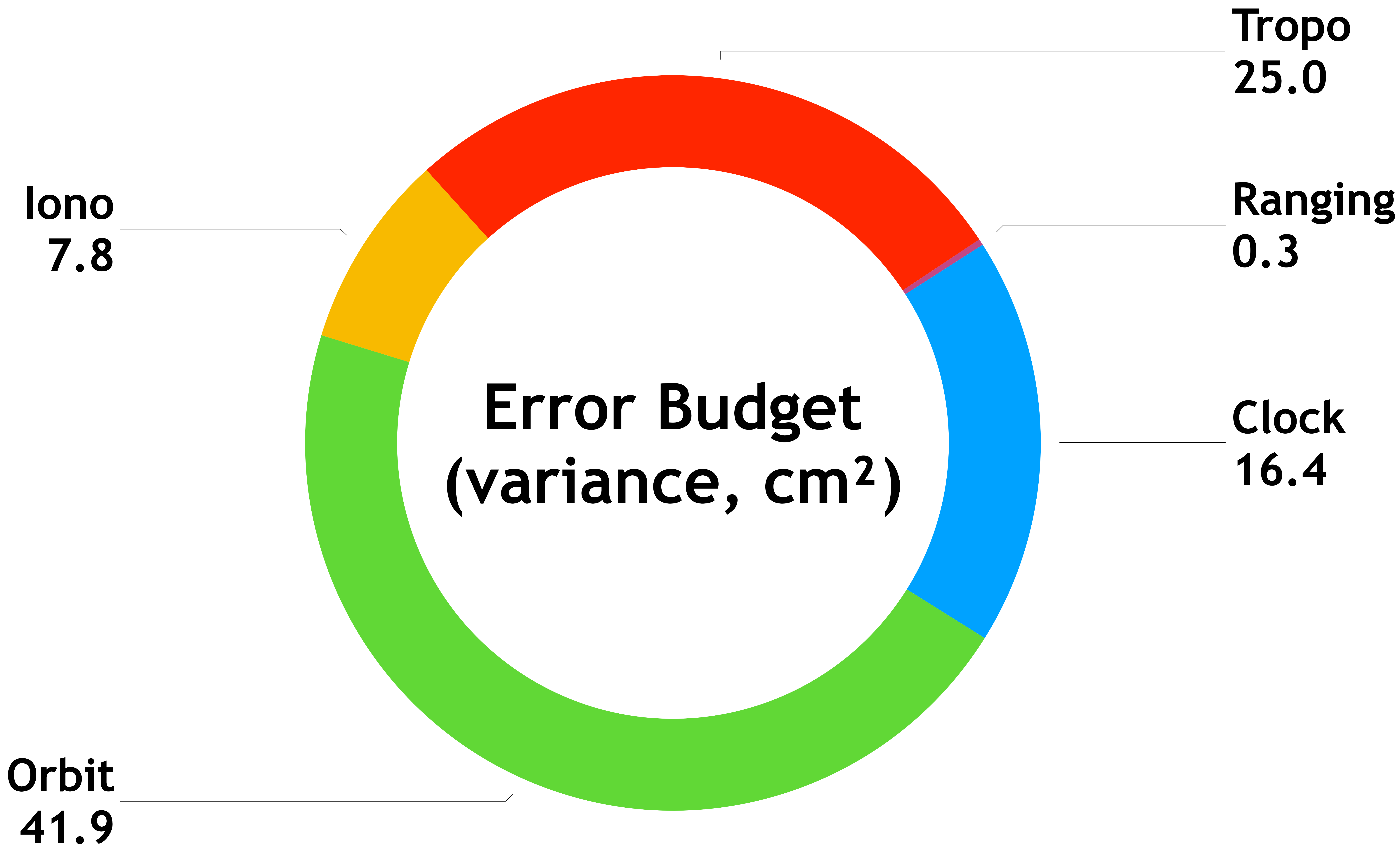
Age-of-Ephemeris = 50s



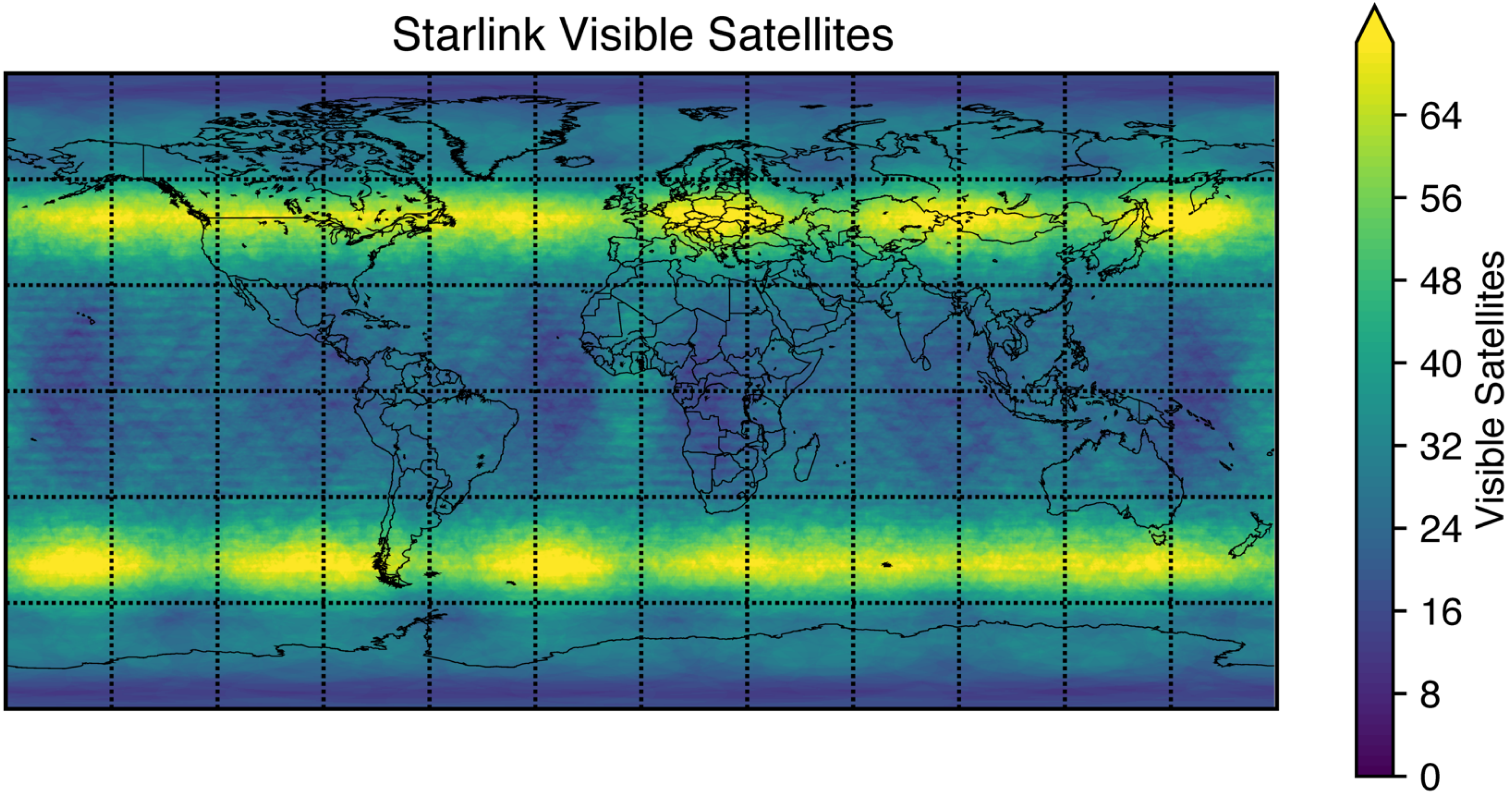
Age-of-Ephemeris = 50s

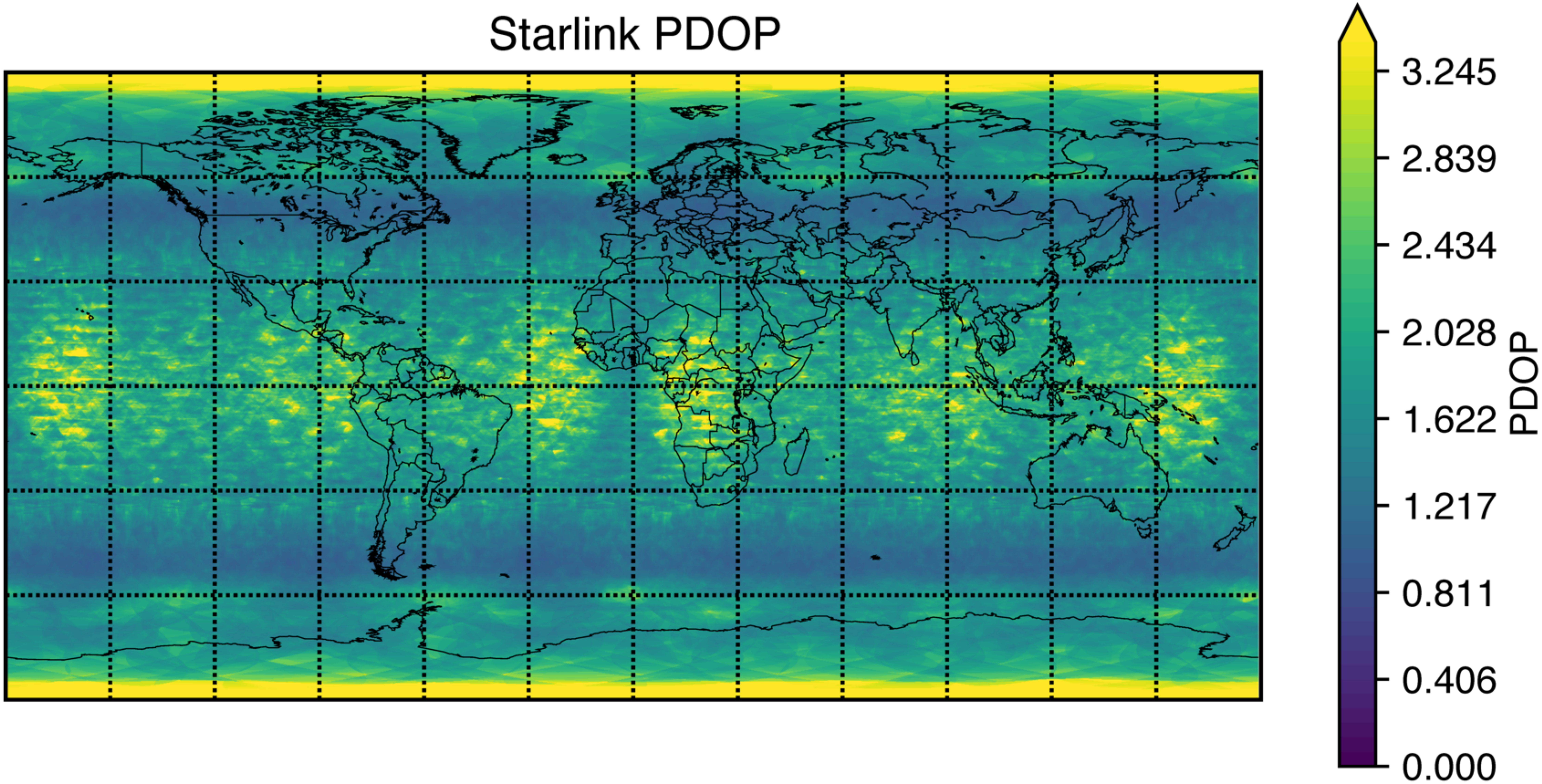


Age-of-Ephemeris = 1s

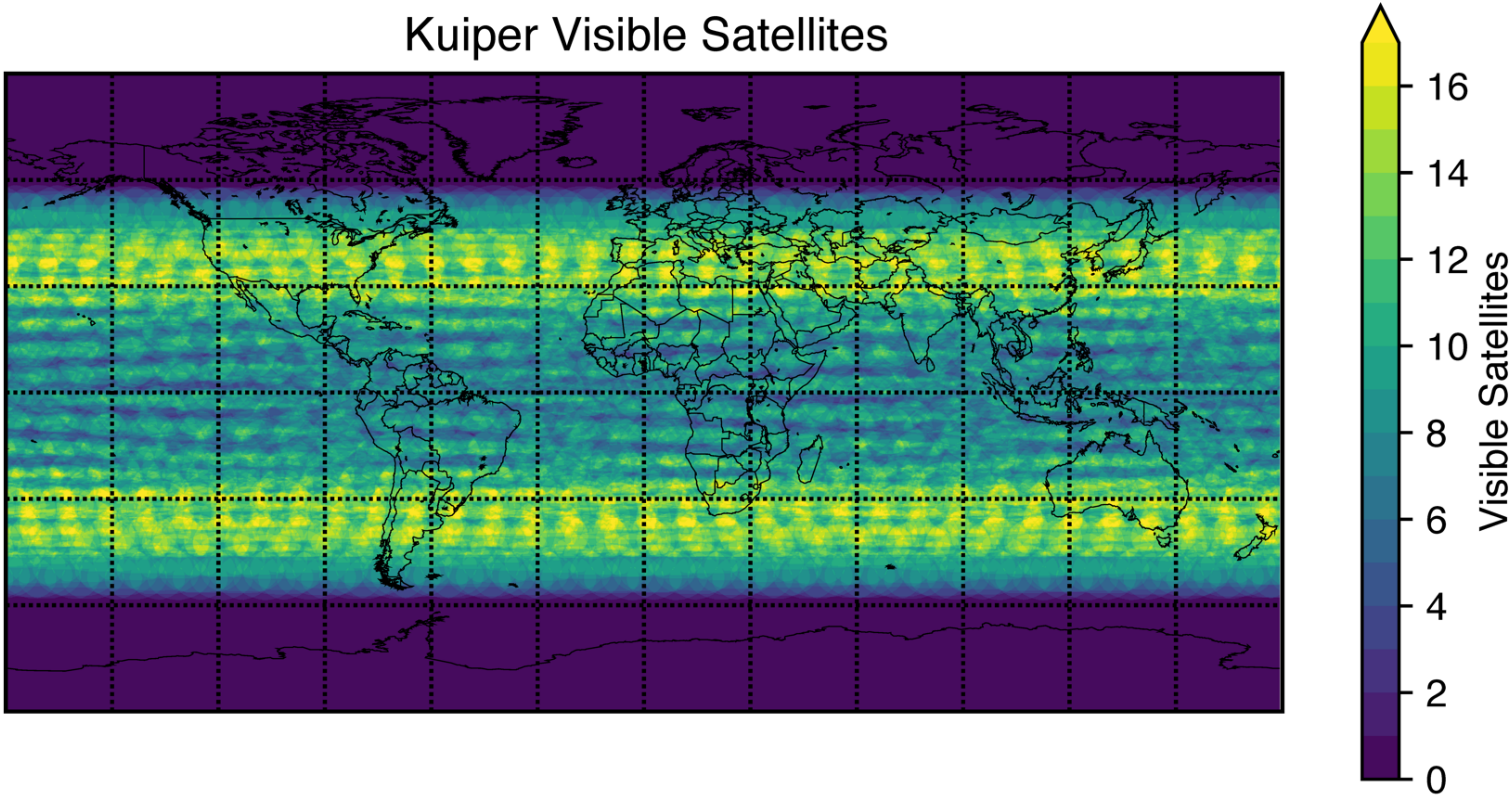


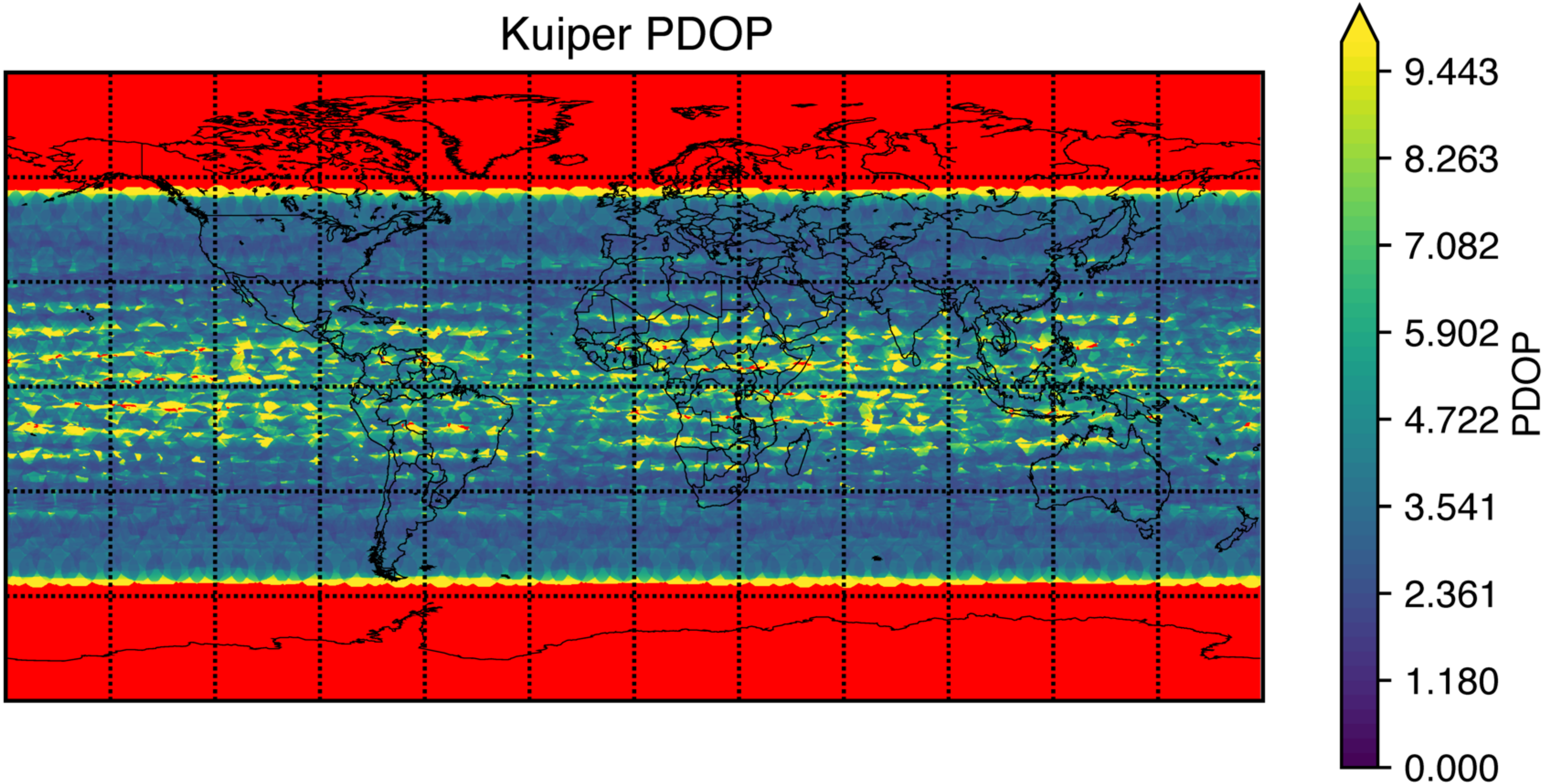
Age-of-Ephemeris = 1s

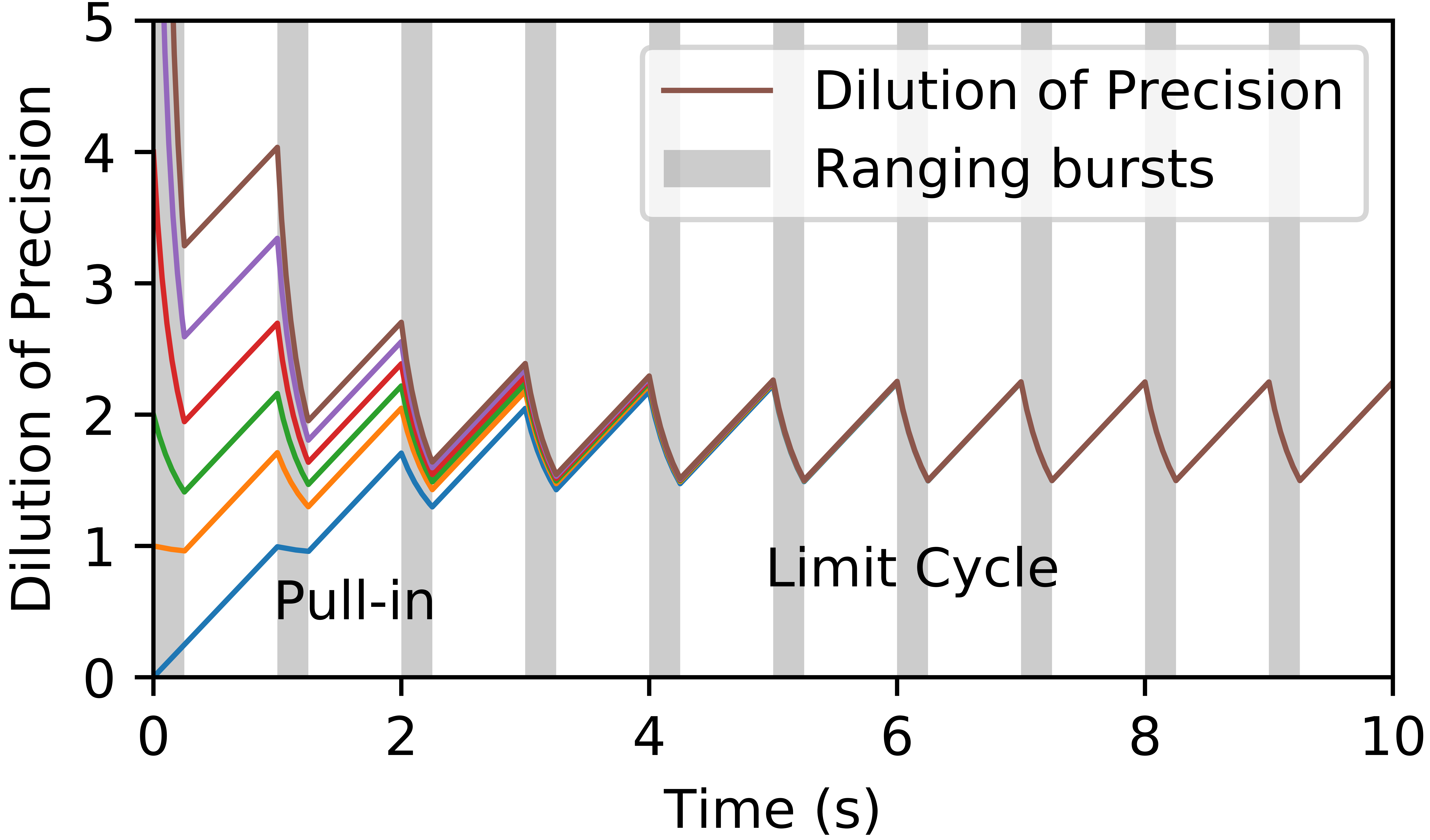

Starlink Visible Satellites
Visible Satellites
0
8
16
24
32
40
48
56
64



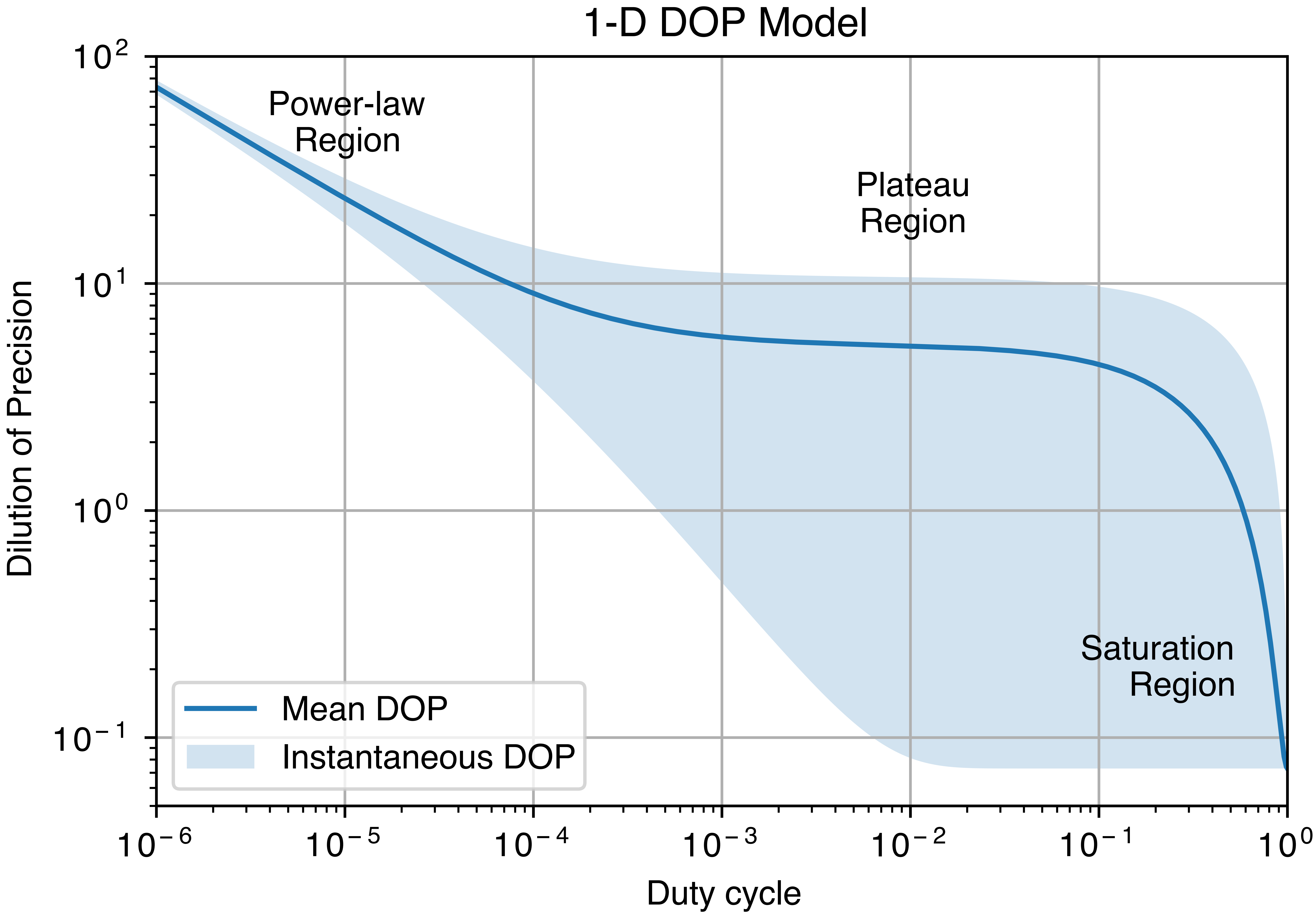
Starlink PDOP
3.245
2.839
2.434
2.028
1.622
1.217
0.811
0.406
0.000
PDOP


Kuiper Visible Satellites
16
14
12
10
8
6
4
2
0
Visible Satellites


Kuiper PDOP
9.443
8.263
7.082
5.902
4.722
3.541
2.361
1.180
0.000
PDOP



Dilution of Precision
Ranging bursts
Pull-in
Limit Cycle
Dilution of Precision
Time (s)
0
1
2
3
4
5
0
2
4
6
8
10


1-D DOP Model
Power-law
Region
Plateau
Region
Saturation
Region
Mean DOP
Instantaneous DOP
Dilution of Precision
Duty cycle
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}

Quantity	Description	Estimate
τ	Ephemeris Update Interval	$50\,\mathrm{s}$
T	Clock RMS error, worst case	$0.271\,\mathrm{m}$
R	Orbit RMS error radial, worst case	$0.037\,\mathrm{m}$
A	Orbit RMS error along-track, worst case	$0.057\,\mathrm{m}$
C	Orbit RMS error cross-track, worst case	$0.049\,\mathrm{m}$
w_R	Geometric weight factor, radial	0.774
w_A	Geometric weight factor, along-track	0.448
w_C	Geometric weight factor, cross-track	0.448
σ_{SISURE}	Satellite timing and positioning errors	$0.301\,\mathrm{m}$
σ_{IONO}	Ionospheric delay error	$0.028\,\mathrm{m}$
σ_{TROPO}	Tropospheric delay error	$0.050\,\mathrm{m}$
σ_{RNM}	Receiver noise and multipath errors	$0.005\,\mathrm{m}$
σ_{URE}	User ranging error per satellite	$0.307\,\mathrm{m}$
$\Delta x_{95,H}$	95% horizontal error	$0.558\,\mathrm{m}$
$\Delta x_{95,V}$	95% vertical error	$0.717\,\mathrm{m}$
Δx_{95}	95% total error	$1.205\,\mathrm{m}$

Age-of-Ephemeris = 50s

$$95\%\ \text{horiz. error} = \sigma_{\mathrm{H}} \cdot \sqrt{F^{-1}_{\chi^2_2}(0.95)},\ \text{where } \sigma^2_{\mathrm{H}} = \mathrm{HDOP} \cdot \sigma^2_{\mathrm{URE}}$$

$$95\%\ \text{vert. error} = \sigma_{\mathrm{V}} \cdot \sqrt{F^{-1}_{\chi^2_1}(0.95)},\ \text{where } \sigma^2_{\mathrm{V}} = \mathrm{VDOP} \cdot \sigma^2_{\mathrm{URE}}$$

$$\sigma^2_{\mathrm{URE}} = \sigma^2_{\mathrm{SISURE}} + \sigma^2_{\mathrm{IONO}} + \sigma^2_{\mathrm{TROPO}} + \sigma^2_{\mathrm{RNM}}$$

$$\sigma_{\mathrm{SISURE}} = \text{Signal-in-Space User Range Error}$$

$$\sigma_{\mathrm{IONO}} = \text{Ionospheric delay uncertainty}$$

$$\sigma_{\mathrm{TROPO}} = \text{Tropospheric delay uncertainty}$$

$$\sigma_{\mathrm{RNM}} = \text{Receiver noise and multi-path}$$

$$\sigma^2_{\mathrm{IONO}} = \left(\frac{40.3 \times 10^{16}\,\mathrm{m/s^2/TECU} \cdot \sigma_{\mathrm{STEC}}}{f^2}\right)^2 \qquad \approx (0.028\,\mathrm{m})^2$$

$$\sigma^2_{\mathrm{TROPO}} = \sigma_{\mathrm{TROPO\ (GPS)}} \qquad \approx (0.050\,\mathrm{m})^2$$

$$f = \text{center frequency} \qquad \approx 12\,\mathrm{GHz}$$

$$\sigma_{\mathrm{STEC}} = \text{Uncertainty in Slant Total Electron Content} \qquad \approx 10\,\mathrm{TECU}$$

$$\sigma^2_{\mathrm{RNM}} \geq \frac{3c^2 k_{\mathrm{B}} T_{\mathrm{RX}}}{2\pi^2 W^2 P\, t_{\mathrm{integrate}}}$$

$$k_{\mathrm{B}} = \text{Boltzmann's constant}$$

$$T_{\mathrm{RX}} = \text{noise temperature}$$

$$= 273\,\mathrm{K} \cdot 10^{0.1 \cdot \text{noise figure}}$$

$$W = \text{bandwidth} = 60\,\mathrm{MHz}$$

$$P = \text{received power}$$

$$t_{\mathrm{integrate}} = \text{ranging burst duration} = 500\,\mu\mathrm{s}$$

$$\text{noise figure} = 6\,\mathrm{dB}$$

$$T = c \cdot \sqrt{\tfrac{2\pi^2}{3} h_{-2}\tau^3 + \tfrac{1}{2} h_0 \tau + \begin{bmatrix} t & 1 \end{bmatrix} \Sigma_0 \begin{bmatrix} t \\ 1 \end{bmatrix}}$$

$$h_{-2} \approx 6 \times 10^{-25}\,\mathrm{s}^{-1}$$

$$h_0 \approx 2 \times 10^{-25}\,\mathrm{s}$$

$$\mathrm{d}\begin{bmatrix} f_t \\ \phi_t \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} f_t \\ \phi_t \end{bmatrix} \mathrm{d}t + \begin{bmatrix} \pi\sqrt{h_{-2}} & 0 \\ 0 & \sqrt{h_0}/2 \end{bmatrix} \begin{bmatrix} \mathrm{d}B_{1,t} \\ \mathrm{d}B_{2,t} \end{bmatrix}$$

$$\begin{bmatrix} f_0 \\ \phi_0 \end{bmatrix} \sim \mathcal{N}(0, \Sigma_0)$$

$$\sigma^2_{\mathrm{SISURE}} \leq (w_R R + T)^2 + w_A^2 A^2 + w_C^2 C^2$$

$$T = \text{RMS clock ephemeris error}$$

$$R = \text{RMS orbit ephemeris error, radial}$$

$$A = \text{RMS orbit ephemeris error, along-track}$$

$$C = \text{RMS orbit ephemeris error, cross-track}$$

$$w_A^2 = \text{Geometric weight factor for along-track errors to contribute to average ranging error}$$

$$= \frac{a^2 + a(u+1) + 1}{8a^2} + \frac{(a^2-1)^2 \log((a-1)^2/(1-2au+a^2))}{16a^3(1-u)}$$

$$w_C^2 = \text{Geometric weight factor for cross-track errors to contribute to average ranging error}$$

$$= w_A^2$$

$$w_R^2 = \text{Geometric weight factor for radial errors to contribute to average ranging error}$$

$$= 1 - 2w_A^2$$

$$u = \frac{\cos^2 E + \sqrt{a^2 - \cos^2 E}\sin E}{a}$$

$$a = 1 + \text{altitude}/r_{\mathrm{Earth}}$$

$$E = \text{elevation mask} = 35^\circ$$

$$c = \text{speed of light}$$

$$R, A, C = \sqrt{\frac{1}{3}\sigma_a^2 \tau_a \left(2(t-\tau_a)^3 - 12e^{-t/\tau_a} t \tau_a^2 + 5\tau_a^3 - 3e^{-2t/\tau_a}\tau_a^3\right) + v^T \Sigma_0 v},\ \text{with}$$

$$v = \begin{bmatrix} 1 \\ t \\ \tau_a(t + \tau_a(e^{-t/\tau_a} - 1)) \end{bmatrix}$$

$$\sigma^2_{a,R} = (100 \times 10^{-9}\,\mathrm{m/s^2})^2 \qquad \tau_{a,R} = 2400\,\mathrm{s}$$

$$\sigma^2_{a,A} = (100 \times 10^{-9}\,\mathrm{m/s^2})^2 \qquad \tau_{a,A} = 2400\,\mathrm{s}$$

$$\sigma^2_{a,C} = (20 \times 10^{-9}\,\mathrm{m/s^2})^2 \qquad \tau_{a,C} = 2400\,\mathrm{s}$$

$$R_0 = 0.034\,\mathrm{m} \qquad A_0 = 0.054\,\mathrm{m} \qquad C_0 = 0.048\,\mathrm{m}$$

$$\mathrm{d}\begin{bmatrix} p_t \\ v_t \\ a_t \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1/\tau_a \end{bmatrix} \begin{bmatrix} p_t \\ v_t \\ a_t \end{bmatrix} \mathrm{d}t + \begin{bmatrix} 0 \\ 0 \\ \sigma_a/\sqrt{\tau_a} \end{bmatrix} \mathrm{d}B_t$$

$$\begin{bmatrix} p_0 \\ v_0 \\ a_0 \end{bmatrix} \sim \mathcal{N}(0, \Sigma_0)$$