Determining the Earth's Gravity Field in the 20th and 21st Century

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Content

- Satellite Orbits
- Orbit and gravity field determination
- The Earth's Gravity Field
- SLR, CHAMP, GRACE, GOCE
- GRACE-FO: GRACE-Follow On

Satellite Orbit

A satellite orbit, or satellite ephemeris, is a table of satellite positions allowing it to interpolate the satellite positions, velocities, etc., to any point in time *t* within the range of the table.

Satellite orbits solve the so-called equations of motion, stating that the acceleration of a satellite equals the sum of the forces acting on the satellite, divided by the satellite mass.

Observations are values of functions of orbital positions, velocities, etc.

Based on the observations, orbit determination results either directly in an ephemeris or in (few) parameters allowing it to generate this ephemeris.

Orbit and Gravity Field Determination

$$\ddot{\mathbf{r}}_j = -GM\frac{\mathbf{r}_j}{r_j^3} + \mathbf{g}_j(t, \mathbf{r}_j, \dot{\mathbf{r}}_j, q_1, \ldots, q_d, q_{j1}, q_{j2}, \ldots, q_{jd'}) \doteq \mathbf{f}_j$$

$$\mathbf{r}_j(t_0) = \mathbf{r}(t_0; a_{j0}, e_{j0}, i_{j0}, \Omega_{j0}, \omega_{j0}, u_{j0})$$
$$\dot{\mathbf{r}}_j(t_0) = \dot{\mathbf{r}}(t_0; a_{j0}, e_{j0}, i_{j0}, \Omega_{j0}, \omega_{j0}, u_{j0}) \ ,$$

The equations of motion for satellite orbits *j=1,2,...* decompose the force field (on the r.h.s.) into the main term of the Earth gravity field (GM-term) and the perurbation terms g_j, which in turn depend on general parameters q_i, *i=1,2,..,d,* and satellite-specific parameters $q_{ji'}$, *i' = 1,2,…,d'.*

Gravity field parameters are general, air drag and radiation pressure parameters are satellite-specific.

The Earth' gravity field

$$V(r,\lambda,\phi) = \frac{GM}{r}\sum_{n=0}^{\infty}\left(\frac{a}{r}\right)^n \sum_{m=0}^{n} P_n^m(\sin\phi)\{C_{nm}\cos m\lambda + S_{nk}\sin m\lambda\}$$

$$\sigma_n^2 = \sum_{m=0}^{n}[C_{nm}^2 + S_{nm}^2] \qquad \Delta\sigma_{n,CS}^2 = \sum_{m=0}^{n}[(C_{t;nm} - C_{r;nm})^2 + (S_{t,nm} - S_{t,nm})^2]$$

The gravitational acceleration due to the Earth acting on a satellite is the gradient =(dV/dx,dV/dy,dV/dz) of the Earth's gravity potential *V*.

n **is the degree of a term,** ***m*** **its order.**

σ_n is the signal strength per degree *n*, $\Delta\sigma_n$ is the so-called difference degree amplitude between two sets of gravity field parameters with subscripts "*r*" and "*t*".

SLR, CHAMP, GRACE, GOCE

The global characterisitcs of the Earth's gravity potential emerge from satellite motion since the advent of the space age.

The first satellite-based gravity fields were obtained from astrometric observations (see SAO Standard Earths).

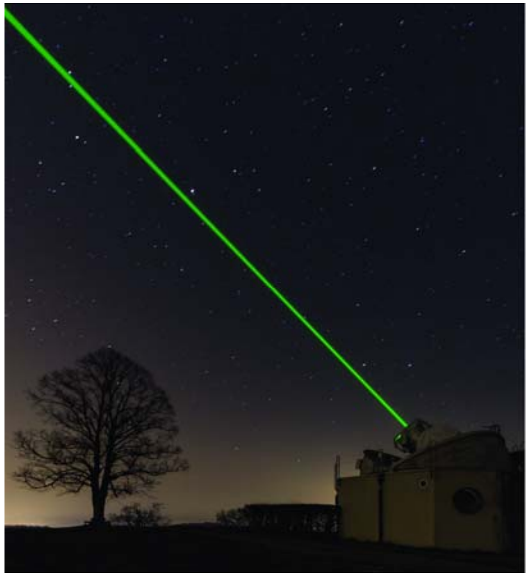
Astrometry was replaced in the 1960s-1970s by SLR, where «cannon ball satellites» were and are observed.

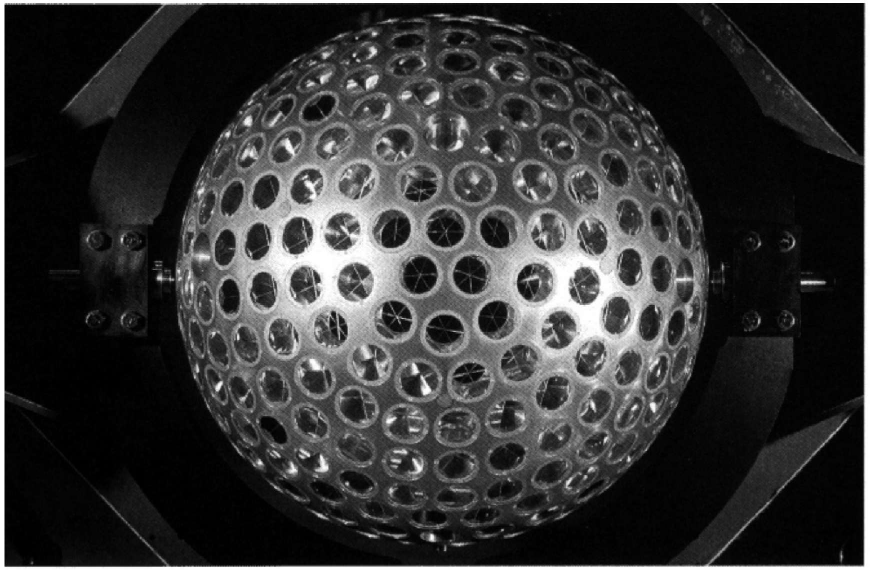
Cannon ball satellites are massive spherical satellites minimizing the effect of non-gravitational forces, like air drag and radiation pressure.

With the advent of the 21st century, gravity field determination is mainly based on dedicated satellite missions.

The CHAMP, GRACE, and GOCE missions are the pathfinders.

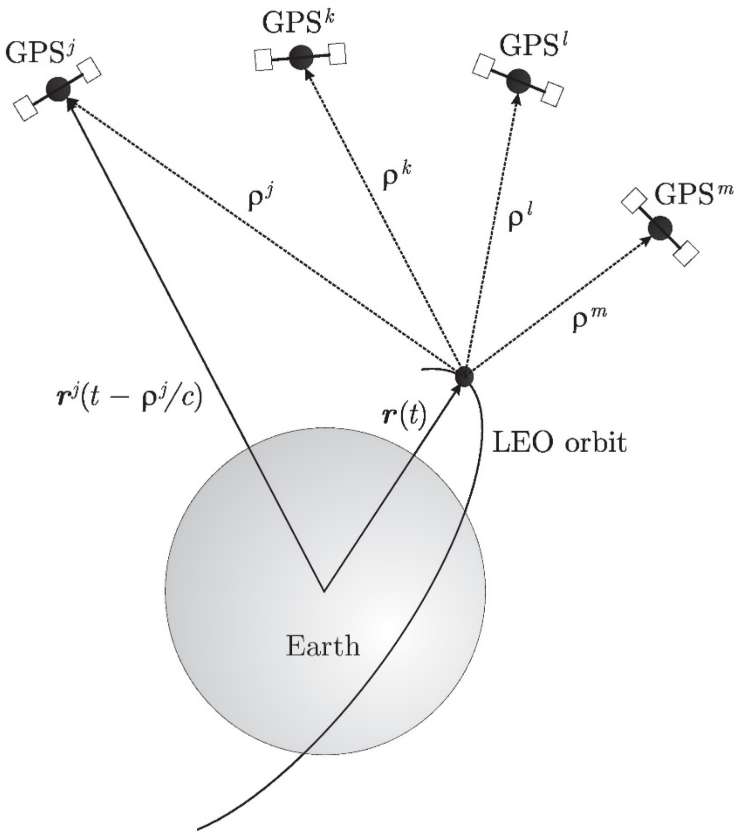
The SLR Era





SLR sends short light pulses from observatiories (e.g., Zimmerwald observatory left) to satellites equipped with retro-prisms (e.g., the Lageos satellites (right)), which reflect the signals to the observatory. The **light travel times (observatory-satellite-observatory) are the observations**.

LEO Orbits with GPS/GNSS



The GPS measurements, gathered by the on-board receiver, may be used to determine a kinematic orbit using PPP.

The kinematic positions serve as pseudo-observations in a generalized orbit determination process to solve for the gravity field and orbit parameters

Many satellite arcs/orbits have to be combined to obtain the parameters of a gravity field.

The GPS observation technique is used for GRACE, GOCE, GRACE-FO in addition to the dedicated observation methods.

CHAMP, GRACE, GOCE Characteristics

	CHAMP	GRACE	GOCE
Height (km)	450 - 300	500 - 450	255 - 235
incl (deg)	87.3	89	96.7
# satellites	1	2 @ 220 km	1
primary obs	GPS	ISD	gradiometer
additional		GPS	GPS
additional	(Accel)	Accel@ CoM	Accel@CoM

Orbits are «close to polar» LEOs.

All satellites carry GPS/GNSS receivers for POD.

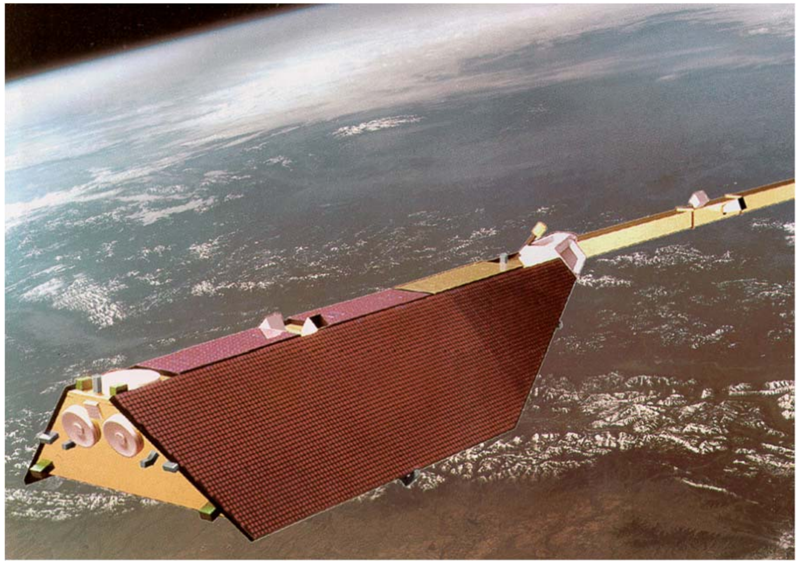
The GOCE gradiometer is an ensemble of 6 accelerometers, mounted on three orthogonal axes, at same distances from satellites CoM

Accelerometer @ CoM measures the non-gravitational forces: In the case of GOCE these accelerations are reconstructed from the accelerations on the axes.

The ISDs between the GRACE twin satellites are measured with μm precision.

Star trackers are on all missions to monitor the orientation of the satellites.

CHAMP



CHAMP was launched in July 2000, the mission ended in September 2010.

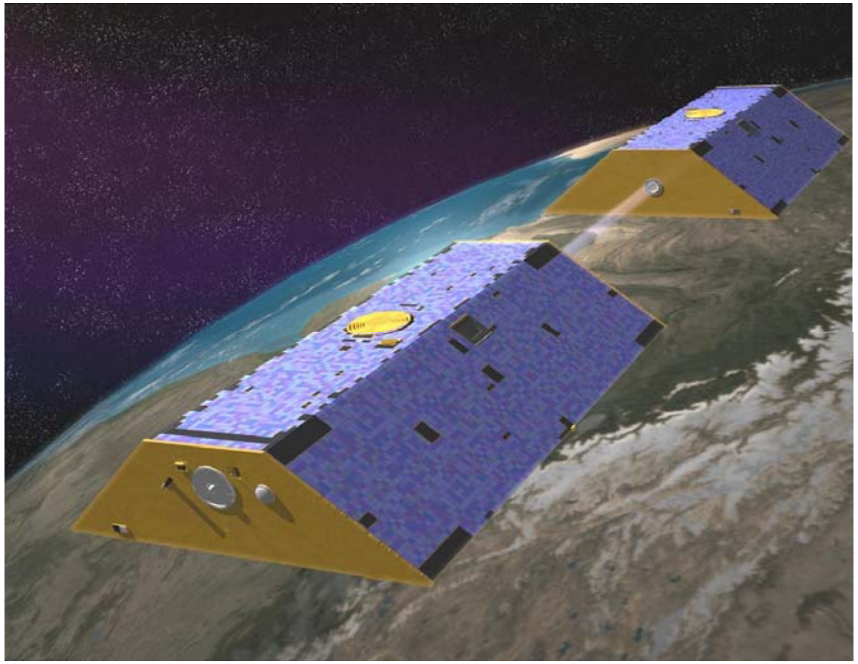
CHAMP was designed to determine the Earth's gravity *and* magnetic fields.

CHAMP also sounded the atmosphere using aft-looking GPS antennas.

The up-looking GPS antenna was used for orbit and gravity field determination

CHAMP was a US/German mission

GRACE



The GRACE mission was launched in March 2002 and ended in Dec 2017 and Mar 2018, respectively.

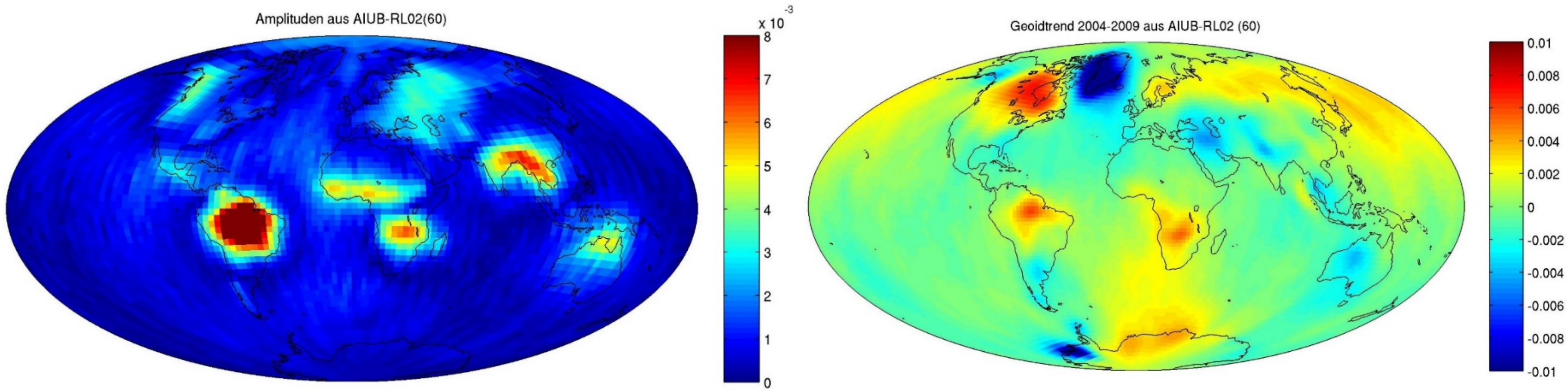
The ISDs between the GRACE twins were measured with µm precision using an ISL in the microwave range.

GRACE allows it to determine a full gravity field up to n x m = 96 x 96 once per month using the ISDs and the GPS measurements.

→ GRACE was designed to determine time variations of the gravity field.

The monthly GRACE gravity fields were compared and combined by EGSIEM to accomodate the needs of the geophysical community.

Time variation with GRACE



Periodic variations (left) and trends (right) of the geoidal heights are key results of the analysis of GRACE monthly fields.

The GRACE mission strongly suggested that the Earth's gravity field should be monitored on a permanent basis → GRACE-FO is a step in this direction.

Illustrations from (Beutler & Jäggi, 2017), for more recent results see, e.g., (Tapley et al. 2019), (Meyer et al., 2019).

GOCE

GOCE was an ESA mission.

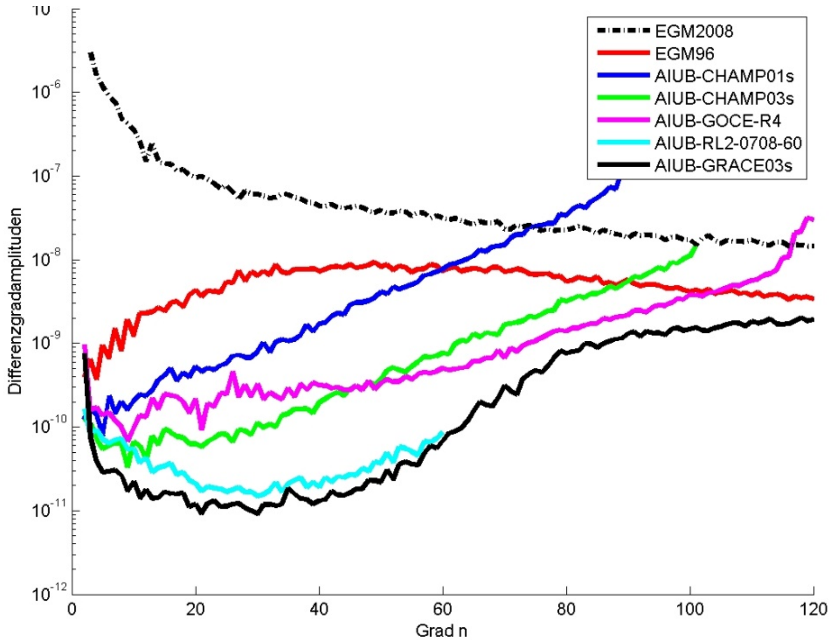
The GOCE gradiometer is a set of six acceleometers allowing it to measure the full gravity tensor ($d^2V/d^2x, d^2V/dxdy, d^2V/dxdz, d^2V/d^2y, d^2V/dydz, d^2V/d^2z$).

GOCE was launched in March 2009 and ended Nov 20, 2013 (extremely low altitude!)

GOCE determined a static gravity field with a resolution of up to about degree *n* & order *m* 300.

The illustration also contains the artist's view of the drag compensation via ion-thrusters.

Performance of Techniques



EGM2008 combines the knowledge of the gravity field in 2008 (satellite+terrestrial) including GRACE, EGM96 the knowledge in 1996 (without the new space missions).

Figure shows the signal strengths σ_n of EGM2008 and the difference degree amplitudes $\Delta\sigma_n$ of other gravity fields w.r.t. EGM2008. y-scale is logarithmic.

One year of CHAMP gravity results (blue) is clearly better than EGM96 (red)!

GRACE-FO

GRACE-FO was realized in the frame of a partnership between NASA, the German Research Centre for Geosciences (GFZ), and the German Space Agency (DLR) – as GRACE was.

GRACE-FO is a «carbon copy» of GRACE, but has in addition an ultraprecise (10-100 times more accurate than the measurements resulting from the microwave link) ISL-system in the optical domain.

GRACE-FO was launched May 22, 2018.

GRACE-FO gravity fields shall be combined by COST-G, an IGS-like global organization.

On May 24, 2019 it was announced that GRACE-FO Level-1 data products are openly available at NASA and GFZ.

→ Science contest is launched!

Frank Webb, the project scientist, is in the best position to brief the PNT board about the GRACE-FO status in fall 2019.

Acronyms

CoM: **Center of Mass**

CHAMP: **ChAllenging Minisatellite Payload**

COST-G: **Combintion Service of Time-variable Gravity field solutions**

EGM: **Earth Gravity Model**

EGM96, **EGM2008**: **Earth Gravity Models based on satellite, terrestrial, aerial methods as available in 1996 and 2008.**

EGSIEM: **European Gravity Service for Improved Emrgency Management**

GOCE: **Gravity field and steady-state Ocean Circulation Experiment**

GRACE: **Gravity Recover and Climate Experiment**

GRACE-FO: **GRACE- Follow-On**

ISD: **Intersatellite Distances**

ISL: **InterSatellite Link**

LEO: **Low Earth Orbit**

PPP: **Precise Point Positioning**

SLR: **Satellite Laser Ranging**

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Acknowledgement/Sources

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Most illustrations stem from (Beutler & Jäggi, 2017), remarks concerning orbit determination theory on (Beutler, 2004, Vol. I, Chap. 8).